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KD-6463
B.E. - II (Sem. IV) (Comp. & IT) Examination
December – 2012
Engineering Mathematics - III

Time : 3 Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **B.E. - II (Sem. IV) (ECC)**

Name of the Subject :

☛ **Engineering Mathematics - III**

☛ Subject Code No. : **6 4 6 3** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) Attempt all questions.

(3) Figures to right indicate the maximum marks of the question.

1 (a) Do as directed : **10**

(i) Prove that :

$$\nabla\left(e^{r^2}\right)=2e^{r^2}\,\bar{r}$$

(ii) Evaluate : $\int_0^1\int_0^{2-x}\int_0^{2-x-y} dz\,dy\,dx$

(iii) Change the order of integration :

$$\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}\,dA$$

(iv) Prove that $f(r)\bar{r}$ is irrotational.

(v) Find $F(0)$ for,

$$f(x)=-\pi,\ -\pi<x<0$$
$$=x,\ 0<x<\pi.$$

(b) Attempt any **three** : **12**

(i) Evaluate $\iint_R \sin\theta\, dA,$ where R is the region in the first quadrant that is outside the circle $r=2$ and inside the cardioid $r=2(1+\cos\theta)$

(ii) Evaluate $\iiint_D t\left(x^2+y^2+z^2\right) dv$ through the volume of the cylinder $x^2+y^2=a^2$ intercepted by the planes $z=0,\ z=h$.

(iii) Find the area that lies inside the cardioid $r=a(1+cas\theta)$ and outside the circle $r=a$.

(iv) Find the volume bounded by the cylinder $x^2+y^2=4$; and the planes $y+z=4$ and $z=o$.

(v) Find the surface area of the portion of the paraboloid $z=x^2+y^2$ below the plane $z=1$.

2 (a) Attempt any **two** : **6**

(i) A vector field is given by $E=\left(x^2+xy^2\right)i+\left(y^2+x^2y\right)j$. Show that E is irrotational and find its scalar Potential.

(ii) Prove that $r^n\,\overline{r}$ is irrotational and is solenoidal when $n=-3$.

(iii) Find the directional derivative of the function $\phi=x^2z+2xy^2+yz^2$, at the point $(1,2,-1)$ in the direction of the vector $\overline{a}=2i+3j-4k$.

(b) Attempt any **two** : **8**

(i) Verify Green's Theorem for the function $E=\left(x^2+y^2\right)i-2xyj;$ and C is the rectangle in the $xy-$plane by $y=0$, $y=b$, $x=0$ and $x=a$.

(ii) Using Stoke's Theorem for the function $\overline{F}=y^2i+yj-xzk$ and ρ is the upper half of the sphere $x^2+y^2+z^2=a^2$.

(iii) Using Gauss' Divergence Theorem to evaluate $\iint \overline{F}\cdot\hat{n}\ \ ds;$ where; where $\overline{F}=4xi-2y^2j+z^2k$ and ρ is the surface bounding the region $x^2+y^2=4;\ z=0$ and $z=3$.

3 (a) Explain Half Range Sine and cosine series. **4**

(b) Attempt any **two** : **10**

(i) Obtain Fourier series to represent $f(x)=\left(\frac{\pi-x}{2}\right)^2$ in the interval $0<x<2\pi$.

(ii) Express $\sin x$ as Cosine Series in $0<x<\pi$.

(iii) Find the Fourier Series for the function

$$f(x)=x, \quad 0<x<1$$
$$=1-x, \quad 1<x<2$$

Deduce that $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}$

4 (a) Attempt the following : **10**

(i) Define Holomorphic function and state necessary and sifficient conditions for $f(t)$ to be analytic.

(ii) State Duplication Formula; Find $\left\lceil 1/4 \right. \left\lceil 3/4 \right.$

(iii) Prove that Beta function is Symmetric. That is $B(m, n)=B(n, m)$

(iv) Prove that $erf(x)+eff\,c(x)=1$

(v) Prove that the function $f(t)=\bar{z}$ is nowhere differentiable.

(b) Attempt any **two** : **6**

(i) Evaluate : $\int_0^1 x^4\left(1-\sqrt{x}\right)^5 dx$

(ii) Prove that :

$B(m,n)=B(m+1,n)+B(m,n+1)$.

(iii) Evaluate : $\int_0^{\pi/2}\sqrt{\cot\theta}\ d\theta$

(c) Solve any **two** : **6**

(i) Evaluate : $\int_0^{\infty}\frac{x^4\left(1+x^5\right)}{(1+x)^{15}}\,dx$

(ii) Prove that : $\int_0^{\infty} x^{2n-1} e^{-ax^2} \, dx = \frac{\overline{|n}}{2a^n}$

(iii) Prove that : $\int_0^{\infty} e^{-(x+a)^2} \, dx = \frac{\sqrt{\pi}}{2} \left[1 - erf(a)\right]$

5 Attempt any **two** : **14**

(a) Determine the solution of the one dimensional heat equation using separable method.

(b) Find the deflection $u(x,t)$ of the vibrating string of length π and ends fixed the corresponding to zero initial velocity and initial deflection $f(x) = k(\sin x - \sin 2x)$, given $c^2 = 1$.

(c) Find the solution of $u_t = c^2 u_{xx}$ together with the initial and boundry conditions

$u(0,t) = u(l,t) = 0$ for all $t \geq 0$ and $u(x,0) = \sin\frac{\pi x}{l}$, $0 \leq x \leq l$.

6 (a) attempt any **two** : **8**

(i) Determine the analytic function whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$.

(ii) Show that the function $u(x, y) - e^x \cos y$ is harmonic. Determine its harmonic conjugate $v(x,y)$ and the analytic function $f(z) = u + iv$.

(iii) Find the bilinear transformation that maps the points $z = \infty, i, 0$ into the points $w = 0, i, \infty$ respectively.

(b) Attempt any **two** : **6**

(i) Evaluate :

$$\int_{t=0}^{1+i} \left\{\left(3x^2 + 4xy + 3y^2\right)dx + 2\left(x^2 + 3xy + 4y^2\right)dy\right\} \text{ along } x = y^2.$$

(ii) Evaluate $\int_0^{2+i} (z)^2 \, dt$, along the real axis to 2 and then vertically to $2+i$.

(iii) Using Cauchy-Integral formula π evaluate $\oint_c \frac{\sin 3z}{z + \pi/2} dz$; where C is the circle $|z| = 5$.